Evaluation of Error Bound for a DT Sliding Mode Control with Disturbance Observer

E. Y. Erdem*, K. Abidi** and A. Sabanovic*

* Sabanci University/Mechatronics Program, Istanbul, Turkey

** National University of Singapore/Electrical and Computer Engineering Department, Singapore

yeganerdem@su.sabanciuniv.edu

g0403709@nus.edu.sg

asif@sabanciuniv.edu

Abstract—**In this paper an estimate of the upper bound of control error for discrete-time implementation of a Sliding Mode Control (DTSMC) combined with disturbance observer is investigated. Having in mind application to PZT high bandwidth actuators and since high accuracy is required the special attention is paid to avoid chattering. Selected structure of proposed SMC controller is proven to offer chattering-free motion. The proposed structure also avoids deadbeat poles that are the cause of large control action which is not desirable in practical applications. The proposed scheme is shown to allow a maximum error bound of O(*T*) for the system with disturbance. The main disturbances are represented by hysteresis and the time variation of the piezo stack parameters. The evaluation of the upper bound of error in such a system is shown and experimentally verified. Closed-loop experiments are presented using the proposed method to verify the theoretical results.**

I. INTRODUCTION

Piezoelectric actuators are ideal for very high-precision motion applications in which the accuracy of positioning is very important and in many cases the closed loop control is the only answer. With development of accurate position transducers the possibility to use robust feedback based nonlinear control methods is becoming an attractive alternative to the model based compensation.

A piezoelectric actuator driven by voltage as input will exhibit nonlinearity between the input (voltage) and output (position). This nonlinearity is mainly due to the parasitic hysteresis characteristics of piezoelectric crystals [1]. Hysteresis characteristic is usually 15-20% thereby greatly reducing performance and its compensation is necessary in order to achieve high accuracy motion. Model based approaches for hysteresis compensation are usually proposed in the literature [2].

Many attempts to develop PZT controller using different control strategies are published. In [3], H∞ based closed-loop control is presented with model based hysteresis compensation. While the method produces good results, it can be made simpler if the hysteresis model based compensation is replaced with a simpler methodology. In [4], a neural-network (NN) based feedforward assisted proportional integral derivative (PID) controller was proposed. A hybrid control strategy using a variable structure control (VSC) is suggested for submicron positioning control [5].

In this paper we aim to design a motion controller for Piezo-Stage having position sensor based on the assumption that the Piezo-Stage can be modeled as a linear lumped parameters (T_{eff}, m_{eff}, c_{eff}, k_{eff}) second order electromechanical system with voltage as the input and position as the output coordinates and hysteresis nonlinearity acting between input voltage and charge. Furthermore it is assumed that the parameters of the model are bounded and have some so-called nominal values (T_N, m_N, c_N, k_N).

In this paper we will be investigating the upper bound of the control error in discrete-time sliding mode controller [6] combined with a disturbance observer in order to achieve high accuracy in the actuator trajectory tracking. By manipulating model of a piezo actuator in a form where nonlinearities due to hysteresis are presented as an additive disturbance acting together with external force to the mechanical system a simple second order observer is designed to estimate lumped disturbance and such, nominal model based structure of disturbance observer, is used here.

This paper is organized as follows. In section II a suitable model of a piezo actuator, based on already known results, is presented and the problem of the discrete-time implementation of SMC is discussed. In the section III the sliding mode based controller and observer design is presented. In section IV the estimation of the upper bound of control error for such a control structure is determined and at the end the experimental results verifying theoretical works are presented.

II. PROBLEM FORMULATION

A. PZT Actuator Modeling

The dynamics of the piezo-stage can be represented by the following second-order differential equation coupled with hysteresis in the presence of external forces

$$m_{eff}\,\ddot{z} + c_{eff}\,\dot{z} + k_{eff}\,z = T(u(t) - h(z,u)) - F_{ext} \tag{1}$$

Here meff denotes the effective mass of the stage, z denotes the displacement of the stage, c_{eff} denotes the effective damping of the stage, k_{eff} denotes the effective stiffness of the stage, T denotes the electromechanical transformation ratio, u denotes the input voltage and $h(z,u)$ denotes the non-linear hysteresis that has been found to be a function of z and u, [2], and F_{ext} is the external force acting on the stage.

Model of a PZT actuator (1) can be represented as the following continuous-time system

$$\dot{x}(t) = Ax(t) + B(u(t) - f(t))$$
$$y(t) = Gx(t) \tag{2}$$

where the state $x(t) \in \Re^n$, the output $y(t) \in \Re^m$, the control $u(t) \in \Re^m$ and the disturbance $f(t) \in \Re^m$ which is assumed to be smooth and bounded. The discrete-time equivalent of (2) can be given as

$$x_{k+1} = \Phi x_k + \Gamma u_k + d_k$$
$$y_k = Gx_k \tag{3}$$

where

$$\Phi = e^{AT},\ \Gamma = \int_0^T e^{A\tau} d\tau B,\ d_k = \int_0^T e^{A\tau} Bf((k+1)T - \tau)d\tau$$

In which d_k is the disturbance and T is the sampling period. From the Taylor series expansion of Γ it can be shown that

$$\Gamma = BT + \frac{1}{2!}ABT^2 + ... = BT + MT^2 + O(T^3),$$
$$BT = \Gamma - MT^2 \tag{4}$$

where M is a constant matrix, [8].

For the proceeding sections the following definition of the order of value will be used: The magnitude of a variable v is said to be of $O(T^n)$ order if and only if $\lim_{T\to 0}(v/T^n) \neq 0$ and $\lim_{T\to 0}(v/T^{n-1}) \neq 0$ where n is an integer and $O(T^0)=O(1)$. Associated with the above definition the following assumption can be made: when the sampling interval is selected to be sufficiently small, for $v_1 \in O(T^n)$ and $v_2 \in O(T^{n+1})$, then $v_1 >> v_2$, the following relations can be derived with proper approximations for $\forall n, m \in Z$ $O(T^{n+1}) + O(T^n) \approx O(T^n)$, $O(T^n) \cdot O(1) \approx O(T^n)$ and $O(T^n) \cdot O(T^{-m}) \approx O(T^{n-m})$ where $\approx$ stands for effective approximation and Z is the set of integers. Based on (4) it can be stated that the magnitude of Γ is $O(T)$.

Control objective is, for the sampled-data system (3), to design a discrete-time sliding mode controller with the sliding surface that is expected to approach the desired value exponentially and the closed loop dynamics of the sampled-data system has its closed loop poles assigned at desired locations.

B. Discrete-Time Sliding Mode Control with Surface Dynamics

Consider the sliding manifold given by

$$\sigma(t) = Cx(t)$$

where $C \in R^{m\times n}$ is a constant matrix of rank m. For a system given as in (3) assume that a sliding manifold is selected as $\sigma = 0$. If the Lyapunov candidate in continuous-time of the system is selected as

$$V = \tfrac{1}{2}\sigma^T \sigma \tag{5}$$

And in order to achieve exponential stability we choose the derivative of the Lyapunov function to be

$$\dot{V} = -\sigma^T E \sigma \tag{6}$$

where E is a constant matrix. If (5) is differentiated and equated to (6) then $\dot{V} = \sigma^T \dot{\sigma} = -\sigma^T E\sigma$. This leads to $\sigma^T(\dot{\sigma} + E\sigma) = 0$. The trivial solution is $\sigma = 0$. Thus, in order to achieve exponential stability, the controller must satisfy $\dot{\sigma} + E\sigma = 0$. From here $\dot{\sigma} = -E\sigma$. With the Euler's approximation, the discrete version is $\sigma_{k+1} = (I - TE)\sigma_k$. We can write $TE=D$, since TE will be calculated as a single parameter $\sigma_{k+1} = (I - D)\sigma_k$. Thus, the sliding surface in discrete-time is defined to be

$$\sigma_{k+1} = (I - D)\sigma_k = (I - D)Cx_k = -\overline{D}x_k \tag{7}$$

which is the approximation of $\dot{\sigma} + E\sigma = 0$, where the error is $O(T)$. It is known that $\sigma_k = Cx_k$ where C is a constant matrix of rank m.

Next step is to calculate the equivalent control. Note that

$$Cx_{k+1} = -\overline{D}x_k \tag{8}$$

and,

$$Cx_{k+1} = C\Phi x_k + C\Gamma u_k + Cd_k \tag{9}$$

If (8) is substituted into (9) the equivalent control can be derived to be

$$u_k^{eq} = (C\Gamma)^{-1}\left[-(\overline{D} + C\Phi)x_k - Cd_k\right] \tag{10}$$

C is selected such that $(C\Gamma)$ is invertible. Since Γ depends on B and T, it changes with the system and the sampling time. On the other hand C can be selected to make $(C\Gamma)$ invertible. If the new control is substituted into (3),

$$x_{k+1} = \left[\Phi - \Gamma(C\Gamma)^{-1}(\overline{D} + C\Phi)\right]x_k - \left(\Gamma(C\Gamma)^{-1}C - I\right)d_k \tag{11}$$

This equation can be written as

$$x_{k+1} = [\Phi - \Gamma K]x_k - \left(\Gamma(C\Gamma)^{-1}C - I\right)d_k \tag{12}$$

Note that in (12) the gain K is given by

$$K = (C\Gamma)^{-1}(\overline{D} + C\Phi),\ \overline{D} = C\Gamma K - C\Phi \tag{13}$$

From (13) it can be seen that the matrix $\overline{D}$ can be calculated from the desired gain K which in tern can be calculated by using any of the standard techniques such as Pole-Placement or LQR, etc. The estimation of the disturbance can be carried on in many different ways. Assuming that system states and disturbance are continuous from (3) one can approximate disturbance d_k by its previous value d_{k-1}, [7, 8].

$$\hat{d}_k = d_{k-1} = x_k - \Phi x_{k-1} - \Gamma x_{k-1} \tag{14}$$

and consequently control becomes

$$u_k = (C\Gamma)^{-1}\left[-\left(\overline{D} + C\Phi\right)x_k - C\hat{d}_k\right] \tag{15}$$

In the proceeding analysis of the closed-loop system, two cases will be studied: (a) when the exact disturbance is included as a feed-forward term of the control input and (b) when the estimate (14) of the disturbance is used instead of the exact value. Before proceeding with the analysis, the following Lemma from [8] will be defined

Lemma: If the disturbance of $f(t)$ in (2) is bounded and smooth, then $d_k = \int_0^T e^{A\tau}Bf((k+1)T-\tau)d\tau$ is approximated by $d_k = \Gamma f_k + \frac{1}{2}\Gamma\dot{f}_k T + O(T^3)$, where $d_k - d_{k-1} \in O(T^2)$ and $d_k - 2d_{k-1} + d_{k-2} \in O(T^3)$.

i) Ideal Case:

By substituting $\hat{d}_k = d_k$ into (15) one can write

$$x_{k+1} = \left[\Phi - \Gamma K\right]x_k + \delta_k \tag{16}$$

where $\delta_k = -\Gamma(C\Gamma)^{-1}Cd_k + d_k$. By substituting estimated value $d_k = \Gamma f_k + \frac{1}{2}\Gamma\dot{f}_k T + O(T^3)$ one can find

$$\delta_k = \left[I - \Gamma(C\Gamma)^{-1}C\right]O(T^3) \in O(T^3) \tag{17}$$

And the solution of the x_k can be given as

$$x_k = \left[\Phi - \Gamma K\right]^k x(0) + \sum_{i=0}^{k-1}\left[\Phi - \Gamma K\right]^i \delta_k \tag{18}$$

Expressing matrix $[\Phi - \Gamma K]$ in the following form:

$$\left[\Phi - \Gamma(C\Gamma)^{-1}\left(\overline{D} + C\Phi\right)\right] = PSP^{-1} \tag{19}$$

where P is the transformation matrix and S is a diagonal matrix, whose elements are the eigenvalues of $\left[\Phi - \Gamma K\right]^k$. Now (18) can be rewritten as

$$x_k = PS^kP^{-1}x(0) + P\left(\sum_{i=0}^{k-1} S^iP^{-1}\delta_{k-i-1}\right) \tag{20}$$

Since $\lambda_{\max} < 1$ for a stable system

$$\sum_{i=0}^{\infty}\|S\|^i = \frac{1}{1-\lambda_{\max}}, \; \|S\| = \lambda_{\max} = \max\{\lambda_1, \lambda_2, ..., \lambda_n\} \tag{21}$$

To find the bound of the states, the limits should be taken.

$$\lim_{k\to\infty}\|x_k\| \le \|P\|\left(\sum_{i=0}^{k-1} S^i\right)\|P^{-1}\|\|\delta_{k-i-1}\| \tag{22}$$

Therefore the limit of states can be calculated as,

$$\lim_{k\to\infty}\|x_k\| \le O(T^{-1})O(T^3) = O(T^2) \tag{23}$$

In the ideal case the error is of the $O(T^2)$ order.

ii) Usage of the Estimated Disturbance

By substituting $\hat{d}_k = d_{k-1}$ into (15) one can write

$$u_k = (C\Gamma)^{-1}\left[-\left(\overline{D} + C\Phi\right)x_k - Cd_{k-1}\right] \tag{24}$$

The state equation with the new controller is

$$x_{k+1} = \Phi x_k + \Gamma(C\Gamma)^{-1}\left[-\left(\overline{D} + C\Phi\right)x_k - Cd_{k-1}\right] + d_k \tag{25}$$

If d_{k-1} is added and subtracted from the equation, it can be shown that

$$\begin{aligned} x_{k+1} = &\left[\Phi - \Gamma(C\Gamma)^{-1}\left(\overline{D} + C\Phi\right)\right]x_k \\ &+ \underbrace{\left(I - \Gamma(C\Gamma)^{-1}C\right)d_{k-1}}_{O(T^3)} + \underbrace{d_k - d_{k-1}}_{O(T^2)} \end{aligned} \tag{26}$$

Similar to the ideal case the limit can be calculated,

$$\lim_{k\to\infty}\|x_k\| \le O(T^{-1})O(T^2) = O(T) \tag{27}$$

Denote that this control is not based on any approximation; the result shown at the end is for worst case scenario.

III. Experimental Work

In order to illustrate the effectiveness of the proposed control experiments are carried out on Piezomechanik's PSt150/5/60 stack actuator connected to SVR150/3 low-voltage, low-power amplifier. The actuator has built-in strain-gages for position measurement. Table I shows the specifications of the piezo-stage. The controller hardware used is the DSPACE DS1102 with the control algorithm executed on MATLAB and SIMULINK with real-time link to DS1102.

TABLE I
PROPERTIES OF THE PIEZO-STAGE

Symbol	Quantity	Value
m_N	nominal mass	9.24e-4 *kg*
c_N	nominal damping	685.0 *Ns/m*
k_N	nominal stiffness	8000000 *N/m*
f	resonance frequency	350 *Hz*
T_N	em transformation constant	3.9 *N/V*

A. Modification for Tracking Tasks

Experiments were carried out for tracking tasks, however, (15) is derived for regulation problem. Hence, it will be necessary to reformulate the control law to deal with the tracking problem. Assuming that the desired state trajectory is r_{k+1}, define the sliding manifold as

$$\sigma_k = C(x_k - r_k) = Ce_k \tag{28}$$

where C is a constant row vector and e is the tracking error. Following the procedure that resulted in (8)

$$C(x_{k+1} - r_{k+1}) = -\overline{D}e_k \tag{29}$$

Substituting (3) and solving for the control, we get

$$u_k^{eq} = (C\Gamma)^{-1}\left[Cr_{k+1} - \overline{D}e_k - C\Phi x_k - Cd_k\right] \tag{30}$$

Since d_k is not available, the controller is modified to

$$u_k = (C\Gamma)^{-1}\left[Cr_{k+1} - \overline{D}e_k - C\Phi x_k - Cd_{k-1}\right] \tag{31}$$

which is the desired controller. Design of matrix $\overline{D}$ is the same as for the regulation task, so, (13) applies to this problem also.

B. Experimental Results

Experiments were conducted on the system using (31). As a reference, a sinusoidal input with 2um peak to peak and 1Hz frequency is given. The controller parameter C is tuned to be more effective on position parameters than on velocity parameters. Figure 1 shows the reference input versus the response. Figure 2 shows the error of the response.

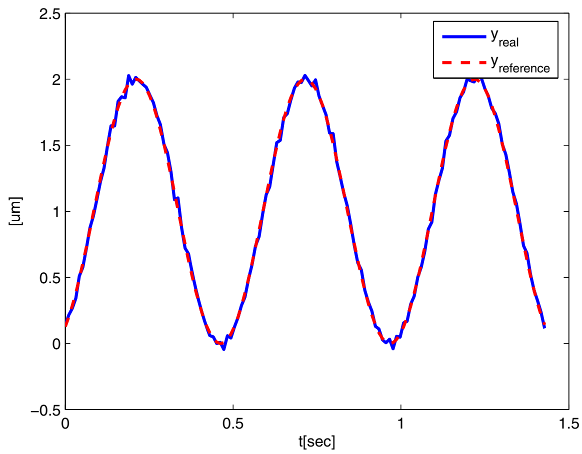

Fig. 1. Closed-loop with compensation response to a sinosoid reference

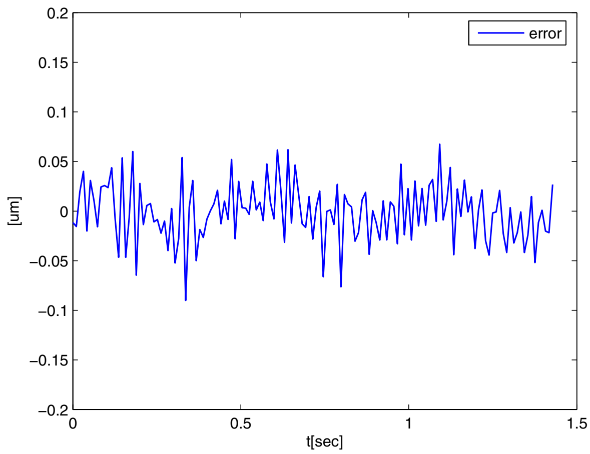

Fig. 2. Error for closed-loop with disturbance observer.

Figure 3 and 4 show the estimated disturbance components. The results show that the proposed controller produces good results.

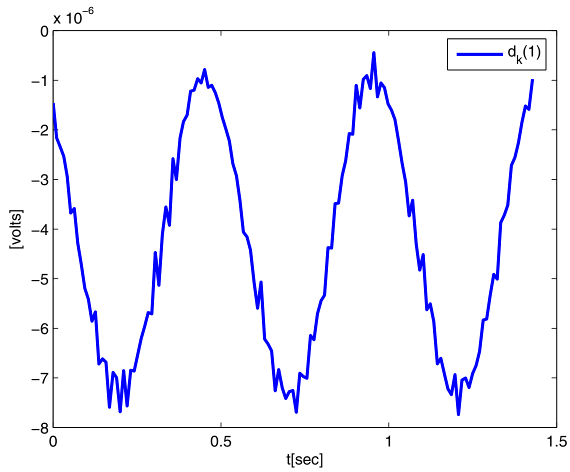

Fig. 3. Estimated Disturbance First Component

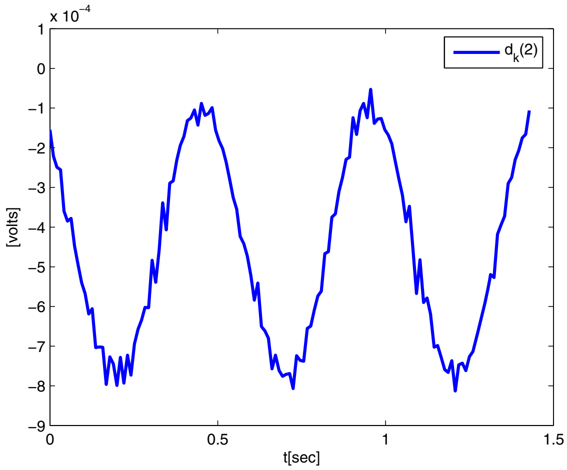

Fig. 4. Estimated Disturbance Second Component

Figure 5 shows the Control Input.

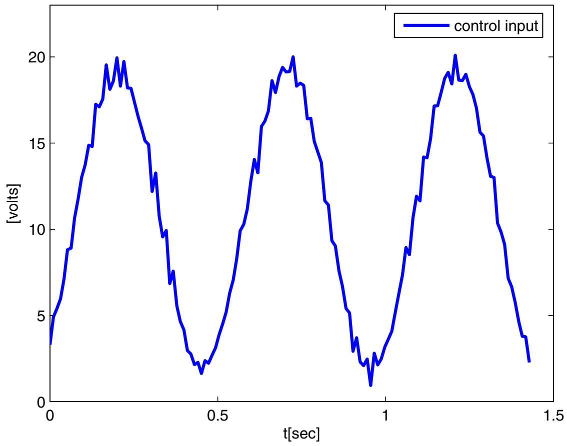

Fig. 5. Control Input.

IV. CONCLUSIONS

In this paper the design of a Discrete-Time Sliding Mode controller based on the Lyapunov is presented. The controller is analyzed for a general system and shown to have very good performance. It was shown that, similar to [10], the zero-order hold causes a limitation on the sliding-mode accuracy. However, it was shown that with partial knowledge of system dynamics it is possible to drive the system within $O(T)$ bound while avoiding the deadbeat response.

REFERENCES

[1] R. Banning, W.L. de Koning and J. M. T. A. Adriaens, "Modeling Piezoelectric Actuators," IEEE/ASME Transactions on Mechatronics, Vol. 5, pp. 331-341, No. 4, 2000.

[2] R. Banning, W.L. de Koning, J. M. T. A. Adriaens and K.R Koops, "State-space analysis and identification for a class of hysteretic systems," Automatica, 37(12), pp. 1883-1892, 2001.

[3] B. M. Chen, T. H. Lee, C.-C. Hang, Y. Guo and S. Weerasooriya, "An H_∞ Almost Disturbance Decoupling Robust Controller Design for a Piezoelectric Bimorph Actuator with Hysteresis," IEEE Transactions on Control Systems Technology, Vol. 7, No. 2, pp. 160-174, 1999.

[4] K. K. Tan, T. H. Lee and H. X. Zhou,"Micro-Positioning of Linear-Piezoelectric Motors Based on a Learning Nonlinear PID Controller," IEEE/ASME Transactions on Mechatronics, Vol. 6, No. 4, pp. 428-436, 2001.

[5] S. B. Chang, S. H.Wu, and Y. C. Hu, "Submicrometer Overshoot Control of Rapid and Precise Positioning," J. Amer. Soc. Precision Eng., vol. 20, pp. 161–170, 1997.

[6] A. Sabanovic, "Sliding Mode Framework in Motion Control– What it Offers?", in the Proc. Of 8th IEEE International Workshop on Advanced Motion Control, AMC'04, Kawasaki, Japan, March 25-28, 2004, pp1-10

[7] Y.-F. Li, "High Precision Motion Control Based on a Discrete-time Sliding Mode Approach", Doctoral Thesis, Royal Institute of Technology, Sweden, Dec. 2001.

[8] K. Abidi and J.-X. Xu, "On the Discrete-time Integral Sliding mode control", IEEE Transactions on Automatic Control (in review)

[9] W.-C. Su, Drakunov S.V., Ozguner U., "An $O(T^2)$ boundary layer in sliding mode for sampled-data systems", IEEE Transactions on Automatic Control, vol.45, no. 3, March 2000, pp 482-485

[10] V. I. Utkin, "Sliding mode control in discrete-time and difference systems", In Variable Structure and Lyapunov Control, Zinober Alan SI, editor [book], Springer-Verlag, 1994, pp 87-107